ABSTRACT
As Part 2 of the redesign numerical efforts of a transonic single stage, the rotor was considered with 3 fundamental design parameters of tailored blading, blade lean and sweep in the similar approach to Part 1 for the stator row. The highly-loaded compressor stage is from a NASA public domain (1969) aiming at a stage total pressure ratio of 1.936 built in radially-stacked MCA blading. Compressor performance was predicted using an in-house CFD code at design speed from deep choke to near-stall flows. Due to extremely high loadings, the rotor experiences strong shocks along nearly entire span, the tailored blading efforts were focused on negative camber in the front and rear-shifted blade thickness to delay shocks and to mitigate their strength through 3 design variants. Upon the best blading design, two positive and negative tangential lean were then applied. In highly-loaded transonic rotor blades, the tangential lean introduces the true sweep toward the tip because of high stagger. A positive tangential-lean case provided considerable performance benefits of a large increase of stall margin and efficiency, thanks to the true forward sweep at the tip pushing shocks more downstream with reduced strength. Although the meridional sweep at the present rotor is not valid for parametric comparisons because of the change of rotor tip diameters, both two cases of forward and backward meridional sweep failed to fit into the baseline flow range. The meridional sweep in transonic rotors was found to be carefully considered to avoid any large changes of choked flows. A case with an additional re-camber at the stator exit for a zero-swirl, was accepted the final stage design when a systematic parameter change of tailored blading, blade tangential lean and meridional sweep was applied to the original rotor and stator blades. Based on CFD predictions, at design mass flow of 83 kg/s, it attained a stage total pressure ratio of 1.952 and a stage isentropic efficiency of 87.3%, relative to original design targets of 1.936 and 84.2%, respectively. If the stall and choke margin are defined with mass flow rates at design speed, the final design provides a 328% increase of stall margin, and a 11% increase of choke margin. It will represent a good design example of highly-loaded transonic front stages with conventional state-of-the-art design philosophy, and will also serve as a reference design for advanced design concept studies to be planned from Part 3.

INTRODUCTION
The application of blade lean and/or sweep has been generally identified as one of the most breakthrough technology in the enhanced aerodynamic design of turbomachinery, especially for axial-flow compressors. In general, the losses from passage/bow shocks and shock interactions with boundary layers and tip leakage flows take a most part of the aerodynamic losses in transonic compressor rotors. As for blade sweep, probably starting from Hah et al. [1] and Wadia et al. [2] many studies have shown that the forward sweep leads to significant benefits. A general agreement has been made on that a forward-swept rotor has higher peak efficiency and a larger surge margin than a radially-stacked one, while a backward-swept one lacks a surge margin. The reduced shock/boundary-layer interaction, resulting from reduced axial flow diffusion and less accumulation of centrifuged blade surface boundary layer at the tip, was claimed as the signature of the forward sweep in transonic rotors. Bergner et al. [3] measured the 3D shock structure and the shock-vortex interaction in two transonic rotors of radially-stacked and forward-swept blading using the 3D Laser-2-Focus...
system. They found the forward-swept rotor had aerodynamic benefits of a reduced tip clearance vortex, a pronounced shock inclination, and an oblique shock front above the midspan.

The influence of blade lean on transonic rotors is much less extensively published than that of blade sweep. Some limited information can be found in a transonic fan of Denton and Xu [4] that the shock in a positively-leaned blade weakens considerably toward the midspan despite being still unswept at tip, while the shock in a negatively-leaned rotor is well away from the leading-edge at tip implying a poor surge margin. However, their modified blades did not represent the blade design the current knowledge on vanes, especially from a case of positively forward sweep and efficiency benefits from leaned stator vanes, especially from a case of positively-leaned hub and negatively-leaned casing. No blade lean was tried in their rotor, though.

The concluding statements in Ji et al. work [7], “blade sweep is a design DOF (Degree of Freedom) that matches the blade element performance along the whole span within entire operation ranges. Its influences on performance are results of trade-off among all the influencing factors”, ensure the need for consideration of other critical design parameters for a better aerodynamic design, such as tailored blading (or blade camber line and thickness distributions), solidity and aspect ratio, etc. Okui et al. [8] demonstrated a 3D optimization of a large design space by changing sweep, tip camber lines and spanwise chord lengths in transonic rotor design. Interestingly they claimed that the highest efficiency gain was obtained with backward sweep and an optimized S-shaped camber line without losing a stall margin. That could support a finding of Benini and Biollo [5] where a backward-swept rotor had a higher efficiency and a larger choke flow than both the datum and a forward-swept design, posing against the general agreement on blade sweep. It can be assumed that when more blade design parameters are involved in transonic rotor design the current knowledge on blade lean and sweep only might need to change.

The present paper, in continuation with the author’s previous work [9], attempts an aerodynamic design study for performance improvements of a single-stage highly-loaded transonic axial-flow compressor, focused on a rotor row only as Part 2. The effects of four design parameters, i.e., blade camber, thickness, lean and sweep, were numerically investigated for a rotor which blades had been radially-stacked, in the same way of Part 1 done for the stator only. The influences of blade solidity and aspect ratio were excluded, which would be another separate topics in future studies.

**NOMENCLATURE**

**LE** : Leading Edge  
**TE** : Trailing Edge  
**x,y,z** : Cartesian coordinates \((\Delta x^2 = \Delta x^2 + \Delta y^2)\)  
**m** : Meridional coordinate \((\Delta m^2 = \Delta z^2 + \Delta r^2)\)  
**LER** : Leading-edge Re-camber  
**t** : Blade thickness  
**β** : Blade camberline (metal) angle  
**θ** : Tangential angle in cylindrical coordinates

**Subscript**  
**b** : Blade  
**n** : Normal  
**1** : Inlet  
**2** : Outlet

*All metal and flow angles are measured from a meridional (or axial) reference.*

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**COMRESSOR STAGE**

The compressor in the present study is a highly-loaded high-Mach-number axial-flow compressor stages in NASA [10][11], which geometry and test results are open to the public. A single compressor stage of 488 m/sec (1,600 ft/sec) tip speed and 838 mm (33 inch) rotor inlet tip diameter was designed to deliver a total-to-total pressure ratio of 1.936 with an adiabatic efficiency of 0.842 at design flow rate of 84.87 kg/sec (187.1 lb/sec) of air. At design point, rotor inlet relative Mach number was designed supersonic over nearly the entire span, while stator Mach number was subsonic across the span with a maximum of 0.89 at hub where the diffusion factor was expected to be 0.6. The stage was without inlet guide vanes, and no swirl was intended at stator exit at all radii. Both 30 rotor and 44 stator blades had radially-stacked MCA sections with a constant chord length from hub to casing. Three-dimensional shape and meridional flow path of the stage are shown in Fig.1. A basic rotor design without any slots was used here. The compressor was tested with uniform inlet flow, but design pressure ratio and efficiency were only met at about 97% of design flow rate at design speed in the test. Accordingly a new design flow rate was set to 83 kg/sec (183 lb/sec) for the present study which stays near the peak efficiency [9].
NUMERICAL METHOD

A single-stage steady-state CFD method [12][13][14], named CNSTURBO and developed by the author, was applied using the compressible Navier-Stokes finite volume method with the 4-step Runge-Kutta time integration scheme and the $2^{nd}/4^{th}$-order artificial dissipation damping. The k-omega equations were implemented as a turbulence closure. H-type structured grids were generated in a multi-block system by solving elliptical Poisson grid equations. In general, the order of 500,000 nodes were used to build rotor and stator domains, respectively. One feature of the numerical grids used is shown in Fig.2. A design tip clearance of 1.27 mm (0.05 inch) was modelled in the rotor, but no clearance was considered in the stator. The conventional mixing plane was applied to the interface between rotor and stator. Neither the fillet radius of blades on the endwalls nor the secondary flow channel through seals under the stator platform was included. At rotor upstream boundary, ambient uniform total pressure and total temperature were given with an axial flow direction, and at stator downstream a hub static pressure was specified to let the radial equilibrium equation determine spanwise static pressure distributions. The solution was regarded as converged when the normalized residual, a measure of local imbalance of continuity, momentum and energy in each conservative control volume, fell below 1.0 x 10^{-5}. It should be noted that the lowest mass flow in predicted compressor maps does not mean a true surge point because any reverse flows occurring at lower flows in the numerical computation become an obstacle to solution stability. The minimum steady flow with an acceptable tolerance of solution convergence would be considered close to the limit of operating ranges. Endwall velocities in figures to be presented hereinafter show non-zero values because they were simply extrapolated from interior cell-centered control volumes for the purpose of an easier post-processing. Some information on the code validation can be found in Part 1 [9].

RESULTS AND DISCUSSION

In Part 1, as a first re-design effort of the compressor, the best stator design had been found (named Lean1 after LER2) from the introduction of both blade lean and new camber styles (i.e., the CDA blading at midspan and the MCA blading at endwalls) to the original. The case will be called “Baseline” hereinafter. The second re-design effort of the compressor in the present Part 2 study is focused on the design changes of Baseline rotor.

1. TAILORED BLADING

1.1 Original Blading

The original rotor blades of Baseline have the MCA blading with a constant chord in the spanwise direction. Depending on approaching Mach levels along the span, the forward section of the MCA was tailored to meet the optimal incidence and the desired throat width. Due to very high loading requirements, however, the rotor cannot avoid strong shocks extending from the tip towards about 10% to 20% span from the hub, as seen in Fig.3 which case was the best redesigned stator in Part 1 study [9]. Any measure to delay such shocks and to mitigate their strength will improve compressor performance including operability.

1.2 New Blading

Rotor camber styles of blade metal camberline angle and blade normal thickness at 3 spanwise sections, i.e., the hub, the midspan and the shroud (or tip), were changed from Baseline to obtain 3 new design cases of RC1, RC2 and RC3, as shown in Table 1 and Fig.4. An interpolated transition from the 3 section changes to the rest spans was smoothly made for total 21 spanwise section geometry. Rear-shifted peak thickness (TM2, TS2) and negative shroud-camber near the leading-edge (CS2) were introduced aiming for a delay of shocks and a reduction of their strength. Higher hub-camber (CH2) was added to compensate for blade loading drops from the negative shroud-camber. The CS2 camber style also has a change of blade inlet metal angle on the shroud for a better incidence control.

1.3 Results and Discussion

Compressor (i.e., rotor + stator) maps, predicted by the present CFD, for those 3 tailored rotor blading were shown in Fig.5, also compared to the original design and Baseline. A
significant increase of compressor isentropic efficiency was observed in RC1, relative to Baseline, which would be credited to the rear-shifted peak thickness style of the rotor blade. The resultant throat opening however brought an excessive choke flow, which led to RC2 design to set back to Baseline choke by closing the blade with CS2 camber. RC2 still kept the same level of efficiency benefits with an acceptable choke flow but lost pressure ratios near design flow due to the negative front camber on the shroud. In RC3 case, a higher loading style on the hub of CH2 was then added to push up pressure ratios similar to Baseline levels while keeping choke flow unchanged. The minimum stall flow of RC3 was a little reduced from RC2 but still larger than Baseline, resulting in a conclusion of tailored blading that RC3 is the most attractive in terms of efficiency, surge/choke margin and pressure ratio.

Table 1 Tailored rotor blading

<table>
<thead>
<tr>
<th>Camberline</th>
<th>Hub</th>
<th>Midspan</th>
<th>Shroud</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>CH1</td>
<td>CM</td>
<td>CS1</td>
</tr>
<tr>
<td>RC1</td>
<td>CH1</td>
<td>CM</td>
<td>CS1</td>
</tr>
<tr>
<td>RC2</td>
<td>CH1</td>
<td>CM</td>
<td>CS2</td>
</tr>
<tr>
<td>RC3</td>
<td>CH2</td>
<td>CM</td>
<td>CS2</td>
</tr>
</tbody>
</table>

Table 1 shows the tailored rotor blading configurations for Baseline, RC1, RC2, and RC3. The thickness distribution for each case is shown in the table, indicating the changes in camber style from baseline to the tailored designs.

More information can be found in the spanwise blade loading distributions of Fig.6 where the 3 blading cases were compared to Baseline at design flow. In RC1, it is clear that shock strengths near the shroud and the midspan on both suction and pressure surfaces were quite reduced compared to Baseline due to throat opening, which contributed to the significant efficiency benefits. A high positive-incidence near
the shroud was gone in RC2 by closing blade inlet, but rotor pressure ratio was decreased near the shroud, relative to RC1, due to smaller loadings by the negative front camber. However the negative front camber pushed both shocks downstream, compared to RC1, implying another benefit in reduction of losses related with shock and boundary-layer developments (see Appendix for details). RC3 shows a recovered blade loading from RC2 similar to RC1 levels, increasing rotor pressure ratio near the shroud, while other aspects remain unchanged from RC2.

2. TANGENTIAL LEAN

Like Part 1 study [9], blade lean and sweep are defined relative to tangential and meridional directions, respectively, for the sake of the author’s convenience in building a blade 3D geometry. Those definitions do not follow the “true” ones which are along a line normal to blade chord and along a blade chord line. Two opposite-lean cases were created on RC3 camber style rotor by changing the stacking line in the tangential direction through shifting their CG (Center of Gravity) locations, relative to the midspan CG, along the span as shown in Fig.7. They show the shape of blade lean locally close to both endwalls by a maximum 1.5 deg-shift in the negative and positive tangential direction, which cases were named RC3L1 and RC3L2, respectively.

Unlike stator vanes, transonic rotor blades get heavily staggered with respect to the axial direction from midspan toward the tip, which requires attention in understanding the tangential lean. In Fig.8 and Fig.9 showing the shape of the leaned rotor blades, it can be seen that the tangentially-leaned tip is also “truly”-swept because of high stagger, while the hub remains tangentially (and probably “truly”) leaned. Although a negative lean has been well reported to fail to improve aero performance from radially-stacked blades, it was added here because both tangential lean and true sweep co-exist for a highly staggered blade.
A first look of the effects of tangential lean, added to RC3 camber, was on the compressor CFD map of Fig.10 where Baseline and the original were also referenced. In RC3L1 (which is with negative tangential-lean and backward true-sweep at the tip), there was a performance degradation from RC3 in terms of all of pressure ratio, efficiency and operability, especially showing a reduction of choke and stall margins. However, RC3L2 (which is with positive tangential-lean and forward true-sweep at the tip) provided considerable performance benefits from RC3 noticeably with a large increase of stall margin and efficiency. The pressure ratio dropped a little near design flow, relative to RC3 and Baseline, but still higher than the original.

![Blade static pressure distributions showing tangential lean effects (at design flow)](image)

**Fig.11** Blade static pressure distributions showing tangential lean effects (at design flow)

Such tangential lean effects can be also found from blade loadings at design flow of Fig.11. The rear-shift of the passage shock was relatively profound at the midspan for RC3L1, while near the shroud for RC3L2. Accordingly RC3L1 lost the rotor pressure ratio from midspan toward the shroud, while RC3L2 limited the loss near the shroud only. In particular, the shock in RC3L2 was significantly delayed on the pressure surface, and became weaker on the suction surface, which would be one of driving factors of the increase of efficiency and stall margin.

![Meridional velocity distributions along the span at design flow at rotor streamwise stations](image)

**Fig.12** Meridional velocity distributions along the span at design flow at rotor streamwise stations (defined in Fig.1)

To investigate more details of the effects of tangential lean, the spanwise distributions of circumferentially-mass-averaged meridional velocity were plotted in Fig.12 at design flow for total 5 streamwise positions whose definitions were described as red lines in Fig.1. In Baseline a strong kink was observed near 60% span at mid-meridional chord (station (b)), resulting from a rapid diffusion behind the shock. In RC3, a smooth velocity profile was brought there without a kink, thanks to the reduction of shock strength, as seen likewise in Fig.6(c). The negative tangential-lean of RC3L1 induced a downward radial force to increase velocities relative to RC3 from the midspan toward the hub all over the streamwise stations, (and the same can be confirmed in Fig.11(a) where the level of both surface pressures dropped from RC3 at 10% span.) At the streamwise station (b) and (c), however, a velocity increase was seen from the midspan toward the shroud, which behaves against the general negative-lean. It would be rather because of the true backward-sweep from the shroud down to the midspan which drops blade loadings towards the trailing edge, (and the same can be seen at 95% span in Fig.11(a)). In RC3L2 a weaker velocity profile was shown near the hub at the station (a) due to upward radial forces by the positive lean.
there. Near the shroud, however, quite a big increase of velocities was found at the station (a) and (b), which implies a rear-shift of blade loadings by the true forward-sweep. Accordingly the shocks were pushed downstream and became weaker.

3. MERIDIONAL SWEEP

Applying the meridional sweep on the present rotor is not straightforward unlike the stator[9], because of two reasons:

(a) Forward or backward sweep will change the enthalpy rise in the rotor due to non-constant shroud (or tip) diameter of the flowpath, losing a valid comparison in the compressor map relative to Baseline.

(b) Due to high Mach levels the change of rotor throat area (and thus choke flows) will be very sensitive to the amount of sweep, shifting compressor map ranges.

![Fig.13 Meridional view of two meridional sweep cases](image)

Two design cases of RC3S1 (forward sweep at the tip) and RC3S2 (backward sweep at the tip) were tried by changing their stacking lines, as shown in Fig.13, despite such considerations. Fig.14 shows their CFD compressor performance, but their ranges of operation failed to fit into the range of Baseline. No improvement was seen in operability, even though there were efficiency gains over Baseline. In fact, blade sections toward the tip were under the true lean because of their high staggers (See Fig.8). No more efforts were added, learning that the meridional sweep in transonic rotors should be very carefully considered to avoid range shifts, and that there are no more significant benefits in aero performance than those from the tangential lean.

4. FINAL DESIGN

In order to fulfill the original design requirement of a zero-swirl at the stage discharge, the re-camber at the trailing-edge of Baseline stator was carried out with RC3L2 rotor design (to be named RC3L2-STER). Fig.15 explains how the process was made in the spanwise direction to get a momentum-averaged swirl of 0.03 deg. The stator re-camber did not affect overall compressor performance as shown in Fig.16, and even a slightly better stall margin was obtained (probably thanks to less flow turning in the stator).

![Fig.14 Compressor maps with the effects of meridional sweep](image)

![Fig.15 Metal and flow angles at stator exit at design flow](image)
CONCLUSIONS

As one of fundamental research topics on aerodynamic design technology of axial-flow compressors, the effects of tailored blading, blade lean and sweep were numerically investigated in a rotor row of a transonic single stage, continued from Part 1[9]. Meaningful findings are summarized below.

TAILORED BLADING

The original MCA blading was well suited for such a high loading duty at transonic flows (i.e., a stage pressure ratio of 1.936). Small efforts were added with a combination of front-negative camber on the shroud and rear-shifted thicknesses (RC3). Some benefits of efficiency and stall margin were found thanks to delayed shocks and reduced shock strength.

TANGENTIAL LEAN

In highly-loaded transonic rotor blades, the tangential lean introduces the true sweep toward the tip because of high stagger. RC3L2 (RC3 with positive tangential-lean) provided considerable performance benefits from RC3 of a large increase of stall margin and efficiency. Near design flow the pressure ratio dropped a little, though. The significant performance benefits come from rear-shifted blade loadings near the shroud by the effects of the true forward sweep, leading to pushing shocks more downstream with reduced strength.

MERIDIONAL SWEEP

Even though the meridional sweep at the present rotor is not valid for parametric comparisons of compressor performance because of the change of rotor tip diameters, both two cases of forward and backward meridional sweep failed to fit into the flow range requirement. The meridional sweep in transonic rotors should be very carefully considered to avoid range shifts from a sensitive change of throat areas.

FINAL DESIGN

The case of RC3L2-STER, with an additional re-camber at the stator exit for a zero-swirl, was accepted the final stage design when a systematic parameter change of tailored blading, blade tangential lean and meridional sweep was applied to the original rotor and stator blades. Based on CFD predictions, at design mass flow of 83 kg/s (which was reduced by the author from the original target of 85 kg/s, because the rig-test[11] had shown an early choked flow), it attained a stage total pressure ratio of 1.952 and a stage isentropic efficiency of 87.3%, relative to original design targets of 1.936 and 84.2%, respectively. If the stall and choke margin are defined with mass flow rates at design speed as,

\[
\text{Stall Margin} = \frac{\dot{m}_{\text{design}} - \dot{m}_{\text{minimum}}}{\dot{m}_{\text{design}}} \times 100(\%)
\]

\[
\text{Choke Margin} = \frac{\dot{m}_{\text{choke}} - \dot{m}_{\text{design}}}{\dot{m}_{\text{design}}} \times 100(\%)
\]

the final design provides a 328% increase of stall margin, and a 11% increase of choke margin. It will represent a good design example of highly-loaded transonic front stages with conventional state-of-the-art design philosophy, and will also serve as a reference design for advanced design concept studies to be planned from Part 3.

REFERENCES

Mach contours near the suction surface of blades in the meridional plane, Fig.A1, provide straightforward insights on the progress of performance changes. Stator redesign [9] removed corner stalls near the endwalls from Original-modified to Baseline. Relative to Baseline, the tailored blading of the rotor in RC3 pushed back the shock wave near the tip, and suppressed the development of shock-boundary-layer growth near midspan. The shock was further delayed near the tip of RC3L2 thanks to positive tangential lean. An additional recamber at the stator exit aiming for a zero-swirl in RC3L2-STER brought a slight change of spanwise Mach at the discharge, but almost identical flow fields were retained while removing stator exit swirls.

Fig.A1  Near-suction-Mach contours at design flow